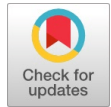


Mathematical Modelling and Analysis to Derive Optimum Brake Pressure for Hill Start Assist System in Commercial Vehicles

Anuj Kumar Shrivastava, Santosh D Dalvi



Abstract: Braking systems are an integral component of both passenger cars and commercial vehicles. Braking lag or delay endangers both the driver and the vehicle, as well as the effectiveness of the brakes. This becomes riskier as the hill begins to rise. To give the driver the chance to run safely when the brakes are applied, the Hill Start Assist is combined with the standard ABS. This can help avoid rollback on an incline. The characteristics that could help enhance this and increase braking efficiency will be the focus of the study. In this research, we design and compute the ideal brake force needed at different angles, and then create a MATLAB model to simulate the outcomes. This would help to make the system more efficient, prevent energy loss, and reduce the dependency on the footbrake valve.

Keywords: Braking, Commercial Vehicle, ABS, Hill Start, brake force, efficiency.

I. INTRODUCTION

Trucks and trailers are examples of commercial vehicles, which are primarily used for load transport. Because of India's terrain, many of the country's highways are built on hills or pass through mountains. In addition to these factors, driving becomes more difficult whether the car is full of passengers or empty, and the roads are slick or damp.

As a result, these cars frequently lose control, leading to accidents. Because braking is such an important phenomena, it necessitates high stability. Commercial vehicles often employ pneumatic air braking as their primary braking system because it provides a powerful braking force while requiring minimum pedal movement. The system is less expensive and more effective than hydraulic systems, and it does not require any special sealing. Commercial vehicles rely on pneumatic systems, particularly of two types: ABS and EBS, for stability. A basic braking system consists of an air compressor, an air dryer, four circuit protection valves, a hand and foot brake valve, load detecting valves, and spring brake actuators [1]. The most common motion combinations that cause accidents in commercial vehicles are jackknifing,

lateral oscillation of the trailer, or a combination of the two. When the articulation angle surpasses the critical limit, jackknifing occurs. Because of the truck and trailer's large relative angular motion, lateral slippage of the rear axle are the primary reason. The second type, or caravan oscillations, are caused mostly by vehicle disturbances. These disruptions cause the vehicle to lose stability, resulting in an accident [2].

Statistical data research reveals that the roll-over is the most harmful and is classified as:

1. Calibrated brake warning devices can help drivers avoid accidents. 3.3%.
2. Avoidable: Proper driving practices and a calibrated brake system may have prevented the accident. 38.4%.
3. Preventable: Despite warnings and expert drivers, 49.7% of incidents were unavoidable. This would necessitate more braking assistance from a tool like HSA.
4. Unpreventable: 8.6% falls outside the scope of the study.

As a result, the findings suggest that about 50% of incidents may have been avoided if warning systems or hill start safety equipment had been available to aid drivers in comprehending the vehicle's proper motion condition [3]. In addition, there is a hill start feature to protect automobiles on an incline. It allows the driver to hold the brake while the pedal is changed, and the automobile is ready to move again. It is apparent that braking has the greatest impact on vehicle stability when compared to other aspects such as suspension control, active steering, jackknifing control, and effective brake control.

A vehicle's braking system is its most important component. The primary factors influencing braking efficiency are braking time and pressure.

II. LITERATURE REVIEW

A. ABS System for the Commercial Vehicles in India

According to 2016 research data, two-wheelers were the most common cause of traffic accidents, followed by commercial vehicles, which accounted for approximately 28% of crashes, including buses and other CVs. Commercial vehicles had the highest fatality rate, accounting for around 25.6%. Indiana Traffic Safety Facts, in partnership with the Indian government, presented similar data for 2017, indicating that accidents involving commercial vehicles climbed by around 5% year on year over the previous five years. The figure below depicts the fatalities and injuries associated with a commercial vehicle accident. With different major criteria to determine the causes of these accidents, some reasons that could be related to the research are dangerous backing and

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Mathematical Modelling and Analysis to Derive Optimum Brake Pressure for Hill Start Assist System in Commercial Vehicles

oversteering on hills or roadways.

The Ministry of Roads and Transportation has stressed safety, and in 2014 it was declared that ABS would be needed on all new automobiles. ABS helps to control automobiles during high-speed emergency braking by freeing the wheels and providing traction control through electrical pressure distribution to the wheels. According to a research, India experienced the most accidents in 2012. Trucks, trailers, and buses account for more than 30% of all accidents. Given that the new models were simple to install, the government issued a circular recommending that they be implemented first.

The relevance of the ABS system was underlined in an essay published in a well-known car industry journal. Additionally, it announced that ABS would be needed for Indian commercial cars beginning April 1, 2015. The two key players listed here are Wabco and Knorr-Bremse. According to OEM research and surveys, customers were willing to spend anywhere between \$80,000 to \$1 lakh depending on the type of car and usefulness.

The training booklet from leading ABS manufacturer Wabco emphasizes the importance of the ASR and ABS features, as well as their progression over time. The document delves into the different features, advantages, and, most crucially, the ABS's basic pieces. The document also outlined how the generic ABS operates. The basic components of any ABS are the compressor, air dryer, air reservoir, four circuit protection valves, foot brake valves, check valves, hand valves, coupling heads (supply and brake), an ABS solenoid valve, and ABS sensors.

B. Market Research Data for the HSA

Hill start assist and downhill assist can help prevent the vehicle from sliding backward while stalled on an incline or beginning from a standstill. Both manual and automatic transmission vehicles will be capable of it. Driving without stress, a longer clutch life, higher fuel economy, accident protection, and so on are only a few of the advantages of HSA.

Sensors used in car hill start aid systems include angle sensors, pressure sensors, torque sensors, wheel speed sensors, electronic control units, and brake actuators. There are two types of transmissions: manual transmission and automatic transmission. Depending on the type of vehicle, it can be classified as either passenger or commercial. Because of increased customer demand for safe and enjoyable driving, the commercial vehicle sector is expected to dominate the hill-start assist device market over the forecast period. The corporation can be divided into four segments: North America, Europe, the Middle East and Africa, and Asia Pacific. It is expected that there will be an increase due to the region's booming market and flourishing automotive sector, especially China, Japan, and India. Additionally, governments are in favor of this because of the new safety standards.

According to Technavio's estimate of the global commercial vehicle hill assist system market from 2017 to 2021, the sector is expected to grow by approximately 8%. As OEMs transfer their operations, the market is expected to rise in APAC as well. Because of tight laws governing the installation of electronic stability systems, developed economies such as the United States, Europe, and Japan

would have the highest adoption rates. The ABS penetration rate in developing economies is showing encouraging progress. The survey also listed the top five commercial vehicle hill assist system market providers: Bosch, Continental, Knorr-Bremse, WABCO, and ZFTRW.

Palkovics, Semsey, and Gerum did research in 1999 on sensor less modifications to the existing EBS system to enable a commercial vehicle roll-over system. The vehicles' motion was categorized into three types: jackknifing caused by uncontrolled relative angular motion, lateral oscillations caused by disturbance, and a combination of the two. The study's findings showed that, while EBS does not totally stabilize a vehicle, adapting the control software to operate with an existing sensor improves roll stability. The EBS platform surely makes it easier to install driving stability controls (DSC) and rollover prevention (ROP) [2].

Honiball and Niekerk published research in 2001 that looked at the test requirements for assessing passenger rollover protection in light commercial vehicles (LCV). The minimal lateral velocity required to cause a rollover was determined using the mass and geometrical features obtained through experimentation. Two types of analysis were used. In the first procedure, the pendulum and canopy were given mass and an initial velocity. How they interacted influenced how the canopy distorted. The second step involved calculating displacement based on the recorded accelerator data. A load was supplied for the FEA investigation to validate the results [4].



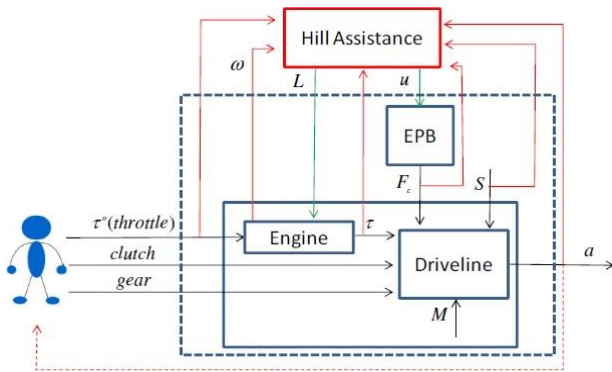
[Fig.1: Sudden Steering Maneuver at Narrow Curve with/without ROP System] [4]

Mass has been shown to be a significant effect in vehicle stability over time. As a result, it was critical to precisely estimate these parameters. In 2009, McIntyre and Ghotikar sought to develop a two-stage Lyapunov-based estimator for vehicle mass and road gradient. The vehicle mass and expected road grade were determined using the least-squares approach. Because the road grade varies over time, a nonlinear estimator is built to provide more precise estimations. The trials were conducted using a number of combinations, and the results showed that they were very close to the generated data, showing the procedure's accuracy. Furthermore, the



data during the braking phase and after the clutch was removed were validated [5].

Dozio and Mandrioli explored a controlled system for the Hill Start aids in commercial cars in 2009 using torque sensing and vehicle movement detection before EPB release. The research was conducted in three stages: the first stage covered vehicle behavior; the second stage covered the detection system; and the third stage covered the control method. The research's limitations included the fact that the findings could only be applied to manual gearbox cars with EPB on steep slopes [6].

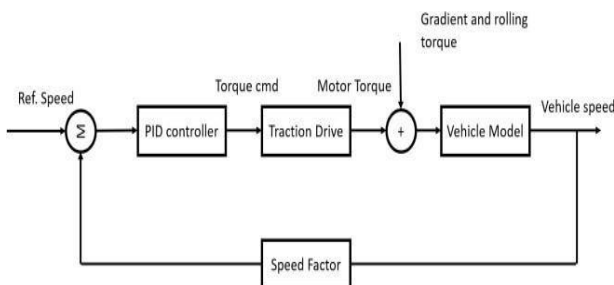


[Fig.2: EPB Control System]

Since most of the research was done on manual gearbox vehicles, it was occasionally difficult to find a similar safe and easy solution for automated vehicles, where the clutch is not involved. In a 2015 research paper, Balasubramanian explains how sensor less hill assistance could be provided by traction control in an electric LCV. The PID controller's modeling and design allow it to determine the holding torque required for the vehicles. Through a controlled loop, information about the vehicle load and slope was transmitted to the PID.

Mathematical models of the rollback force, which took into account changes in the mass factor, were also created during the study. The simulative tests were conducted using MATLAB, and the accuracy of the findings was further verified.

By avoiding the usage of extra sensors, the control system lowers overall costs [7].



[Fig.3: Controlled Model Hill Assist Control System]

Due to the need to strengthen with a safety-related goal, researchers from the Volkswagen Group Brazil carried out a similar study in 2015. The study focused on the Hill Start Aid System for Commercial Vehicles with Automated Gearboxes. In order for the drive axle's service brakes to be controlled by magnetic valve activation and the rest of the vehicle's service brakes to be indirectly controlled by pneumatic logic, the research needed a standard ABS ECU.

It's possible that the method proposed by Carlos and Nuss de Souza works by working with the ABS to precisely apply braking force to the wheels without the driver having to depress the brake pedal. The solution makes it independent of brake temperature, vehicle design, and slopes. The solution makes it independent of brake temperature, vehicle design, and slopes [8].

In 2019 and 2020, Peng, P., and Wang, H. carried out a thorough investigation with a focus on commercial vehicle hill start systems. The first article explained how to build a logic threshold control framework to improve the quality of the hill-start assistance. The solenoid valve was controlled by varying the pulse width and frequency. The framework only made use of the EPB system. The only fixed angles that were recommended were 8°, 13°, and 20° [9].

After that, the study took into consideration a two-layer hill aid system. The amount of pressure required at the slope would be calculated by the first layer and compared to a threshold. The second layer was used to estimate mass online. In this case, a car with an EPB was necessary. Both simulations and real-world tests were used to evaluate the proposed controller, and the outcomes were deemed satisfactory.

There were several formulae for the force applied on the vehicle depending on factors like mass and gravity's acceleration, which were as follows

Different equations for the force acting on the vehicle were determined by considering factors such as mass and acceleration due to gravity [10].

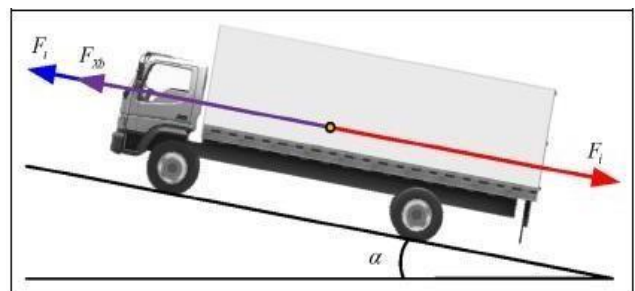
$$F_{\mu} = F_i - F_t \dots (i)$$

Where F_{μ} is the parking brake force, F_i is grade resistance force and F_t is tractive force.

$$F_i = mg \sin \alpha \dots (ii)$$

$$F_t = T_{tq} i g i_0 h / r \dots (iii)$$

$$\text{Desired pressure } Pd = \left(F_{\mu \max} - mg \sin \alpha + \frac{T_{tq} i g i_0 \eta}{r} \right) \frac{P_{rel}}{F_{\mu \max}} \dots (iv)$$



[Fig.4: Forces Influencing Vehicle Longitudinal Dynamics in Hill Start]

In 2012, studies on AMT-based hill start control for large trucks were carried out [11]. When a push button is added, the driver must press it as soon as the vehicle starts to climb the hill [12]. The control method would change the gear to the proper one by means of



Mathematical Modelling and Analysis to Derive Optimum Brake Pressure for Hill Start Assist System in Commercial Vehicles

pneumatic actuation [13]. As soon as the driver releases the hand brake, the air is ready to pass through the ECU-controlled ABS valve [14]. Because the ABS valve is not operating to supply air to the braking chamber, the wheel locks up and rolls back when the motor's output torque drops below the threshold.

III. RESEARCH GAPS

Following were some significant research gaps that could be found through the literature review:

1. The EBS platform was the only one where most of the work was done because it served as the foundation for

- new modules and functionalities like hill start assist systems.
- The LCV was the subject of extensive research, but trucks with trailer-carrying loads were not considered.
 - The majority of the parameters used in the hill start assist system were mass, angle, or road grade; however, the combination of these parameters was not taken into account.
 - The consideration of vehicle type and engine torque was absent.
 - The majority of researchers concentrated on the European market, while the commercial vehicle models from APAC countries were not taken into consideration.

Table 1: Research Gap Table

No	Research Paper Title/ Publication year	Research Summary	Research Gap
1.	Roll-over prevention system for commercial vehicles– additional sensor less the function of the electronic brake system. Vehicle System Dynamics The year 1999 [2]	Enhances the efficiency of the ABS and the comfort of the driver to increase braking functionality. Does not need an additional sensor and is possible by changing the control SW.	The market across Europe, where EBS is widely utilized purely dependent on the SW and only provides rollover protection.
2.	The development of a test specification to determine the rollover protection of passengers in light commercial vehicles fitted with canopies. Accident Analysis & Prevention The year 2001 [4]	The limited lateral velocity that leads to a rollover was calculated using the mass and geometric features that were obtained through experimentation.	Since only ton vehicles were taken into consideration when developing the proposed test solution, it needs to be validated at various levels. restricted to only LCV used for passenger transportation.
3.	A two-stage Lyapunov- based estimator for estimation of vehicle mass and road grade. IEEE Transactions on vehicular Technology The Year 2009 [5]	The study concentrated on a two-stage estimation method for estimating the mass and road grade of heavy-duty vehicles. We employed the least-squares estimate method.	Useful only for calculating vehicle mass and road grade. The angle and other factors were not taken into consideration.
4.	A control system for hill start assistance for commercial vehicles. The year 2009 [6]	A control method has been implemented based on the detection of the torque gearbox from the engine to the wheel.	Suitable only for vehicles with manual transmissions and EPB The suggested approach had problems on low slopes and was only useful for high hills.
5.	Sensor-less hill-assist using traction control in electric. LCV The year 2015 [7]	Through a test of constant torque and acceleration, vehicle parameters were discovered. using mathematical modelling to determine the rollback force as the mass changes the development of a control system algorithm prevents the requirement of extra sensors	Could only be used in commercial electric vehicles. Only at 4 degrees were simulation findings tested on a vehicle
6.	Improved hill starts aid system for commercial vehicles equipped with automated transmission. The year 2015 [8]	A system is suggested that can interact with ABS to deliver the desired amount of braking pressure to the wheels without the driver depressing the brake pedal.	Decreased modulator valve life makes it independent of slopes, vehicle configuration, and brake temperature, which may be difficult given the topographical conditions in India.
7.	Research on the Hill Start Assist of Commercial Vehicles Based on Electronic Parking Brake System The Year 2019 [9]	Created a logic threshold control architecture to enhance the effectiveness of the hill-start assist. The pulse width and frequency modulation are used to control the solenoid valve.	Only the 8%, 13%, and 20% were proposed for the controller. The EPB system was the only thing the framework was dependent on.
8.	Two-layer mass-adaptive hill start assist control method for commercial vehicles. The year 2020 [10]	The first layer determines the hill's needed pressure and compares it to the threshold. Online mass estimation is done using the second layer.	Could only be used if the vehicle has an electronic parking brake.

IV. SYSTEM DESCRIPTION

Assumptions:

1. Equally, Disturbed Mass of Vehicle.

- Assuming Rear 2WD Vehicle.
- Hydraulic Braking System (Braking Pressure is equally distributed).
- Hill Hold Time Interval is assumed as 3 secs

Table 2: Input & Output Parameters

Input	Output
Kerb Weight of Vehicle (Newton)	Deceleration (m/s ²)
Pay Load Weight of Vehicle (Newton)	Inclination Angle (θ) $\tan \theta = \sigma/100$
Initial Speed of Vehicle (m/s)	Braking Force (Newton) = $F_b = v^2 m \cos \theta / 2d$
Gear Ratio(i) = Z_e/Z_s	Braking Distance (m)
Coefficient of Rolling Resistance (μ) = $\mu R = F/G$	Stopping Distance (m)
Slope Grade (σ) = $(\text{rise} / \text{run}) \times 100$	Human Perception Distance (m)
Human Perception Time (secs)	Human Reaction Distance (m)
Human Reaction Time (secs)	Brake Lag Distance (m)
Brake Lag (secs)	--
Hill-Hold Time Interval (secs)	--
Engine Torque (Nm) = $5252 \times \text{HP/RPM}$	--

• Kerb weight of vehicle:

The kerb weight is the vehicle's total weight when it is empty of all passengers and other loads.

• Pay Load Weight of Vehicle:

The maximum weight that a vehicle can safely carry is known as the payload. All the weight inside the cabin and bed makes up a truck. The weight of the trailer pressing against the trailer hitch also referred to as tongue weight, is included in the payload when towing a trailer.

• Braking Force:

The total amount of force applied to a moving body is referred to as braking force, also known as braking power to put an end to it.

• Braking Distance:

The distance a car travels after applying all of its brake pressure until it comes to a complete stop is known as the braking distance.

• Gear Ratio:

Teeth are used to count gears. The gear ratio or speed ratio refers to the relationship between the number of teeth on the follower and the number of teeth on the driver.

• Coefficient of Rolling Resistance:

The force restraining motion when a body rolls on a surface is known as rolling resistance, also referred to as rolling friction or rolling drag.

• Stopping Distance:

This distance represents the total of the various distances the car traveled while the driver made a choice, applied the brakes, and brought the car to a stop.

• Slope Grade:

A road's slope, also known as gradient, is the tangent of the angle formed by its surface and the horizontal.

• Human Perception Distance:

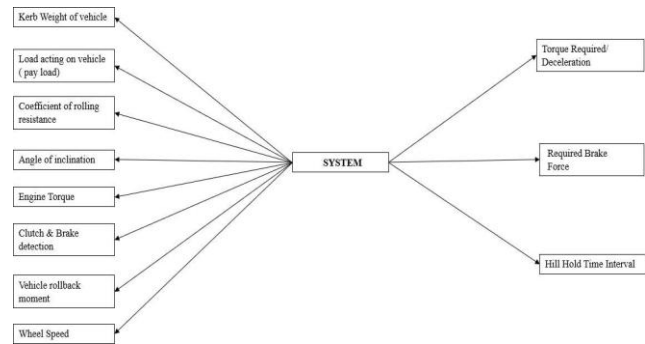
The human perception distance is the distance a car travels before the driver decides to slow down or stop the car after spotting a hazard.

• Human Reaction Distance:

The distance a car travels before the driver decides to stop it after spotting a danger is known as the human reaction distance.

• Brake Lag Distance:

The period after depressing the brake pedal that is needed to apply the brakes.

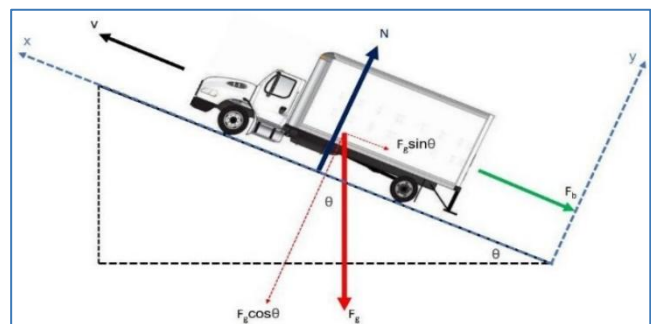


[Fig.5: Input & Output Parameters Block Diagram]

V. FORMULA DERIVATION

When a vehicle is driving up the hill, there are various forces that act on the vehicle.

The driver generates the driving force along the ramp upward by adjusting the gear and throttle. In order to assist the driving force in maintaining the vehicle's static state and preventing it from rolling back, the direction of the braking force goes up along the slope before the driving force overcomes the resistance. A general diagram and the forces acting on the vehicle as it climbs the hill are shown in Figure below. The specifications and information are listed below



[Fig.6: Forces Acting on the Vehicle while Climbing]

A. Derivation for Braking Force

Frictional Force (Braking) is described with the relation.

$$F_b = f \cdot N \dots (1)$$

f...coefficient of static friction.



Mathematical Modelling and Analysis to Derive Optimum Brake Pressure for Hill Start Assist System in Commercial Vehicles

We see that the friction force depends on the vertical force N acting between the wheels and the road. This force will be different on a horizontal surface and a slope.

$$F_b = f \cdot N \quad \dots (2)$$

From Relation,

$$f = m \cdot v^2 / 2 \cdot g \cdot d \quad \dots (3)$$

Force acting on vehicle going uphill: F_g = Gravitational Force
N = Normal force acting on a vehicle.

F_b = Braking force Equation of motion of a vehicle moving upward

d: stopping distance in m.

$$F_b + F_g + N = m \cdot a \quad \dots (4)$$

From diagram,

$$F_{gx} = F_g \cdot \sin \theta \quad \dots (5)$$

$$F_{gy} = F_g \cdot \cos \theta$$

On a slope it holds that:

$$F_b = f \cdot N \quad \dots (6)$$

We express force N1 from relation and substitute for it:

$$F_b = f \cdot F_g \cos \theta \quad \dots (7)$$

We substitute for f from relation: Formula for Braking Force:

$$F_b = v^2 \cdot m \cdot g \cdot \cos \theta / 2 \cdot g \cdot D \quad \dots (8)$$

$$F_b = v^2 \cdot m \cdot \cos \theta / 2 \cdot D$$

B. Derivation of the Generated Torque

The torque created by the pistons within an engine as they reciprocate up and down on the engine's crankshaft, causing it to rotate (or twist), continuously. This torque is then transferred to the vehicle's wheels through the transmission and drivetrain.

Generated Torque is given by.

$$5252 \cdot (\text{HP}/\text{rpm}) \quad \dots (9)$$

Where.

5252 - Is a constant that aligns the units

HP- The horsepower of the specific vehicle on which the test is conducted.

The number of revolutions per minute generated by the vehicle at that moment is the RPM.

First, we need to calculate the climbing resistance.

$$F_a = G \cdot \sin(\alpha) \quad \dots (10)$$

G= vehicle weight + loading in N

α = slope angles in degree

The Climbing torque M_a :

$$M_a = F_a \cdot R_L = G \cdot \sin(\alpha) \cdot R_L (\text{Nm}) \quad \dots (11)$$

R_L : wheel tire radius(m)

For example, a slope is 12 degrees, a vehicle is 50kg, the loading is 75kg, wheel size 18inch.

$$G = (50\text{kg} + 75\text{kg}) \cdot 9.8\text{N}/\text{kg} = 1225\text{N}$$

$$R_L = 0.5 \times 18'' = 0.228 \text{ (m)}$$

$$\text{Torque } M_a = 1225 \times \sin 12^\circ \times 0.228 = 57.81 \text{ (Nm)}$$

Considering there will be other resistance, so the motor Torque M should be.

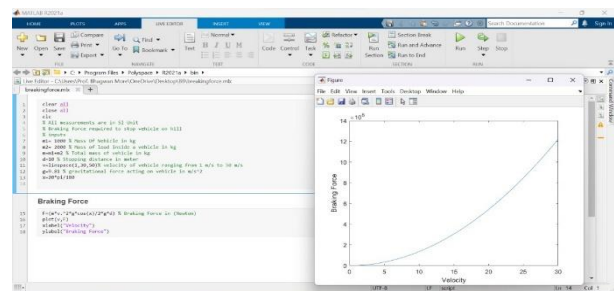
$$M > M_a = 57.8\text{Nm}$$

C. MATLAB SIMULATION FOR BRAKING FORCE

For 20-degree angle of inclination –

Assuming $\theta = 20$ deg, $v = 1$ to 50 m/s, $d = 10\text{m}$, $m = 3000\text{kg}$

$$\therefore F_b = v^2 \cdot 3000 \cdot \cos(20) / 2 \cdot 10$$



MATLAB Simulation for Braking Force at 20 Deg.

For 15-degree angle of inclination –

Assuming $\theta = 15$ deg, $v = 1$ to 50 m/s, $d = 10\text{m}$, $m = 3000\text{kg}$

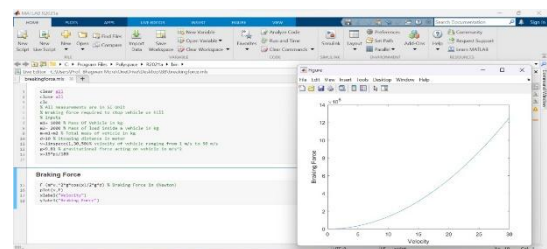
$$\therefore F_b = v^2 \cdot 3000 \cdot \cos(15) / 2 \cdot 10$$

D. MATLAB Simulation for Generated Torque

For generated torque, $g = 16020$, $a = 1-25$, $R_L = 0.508\text{m}$, $HP = 180$ rpm range = 1500-2000

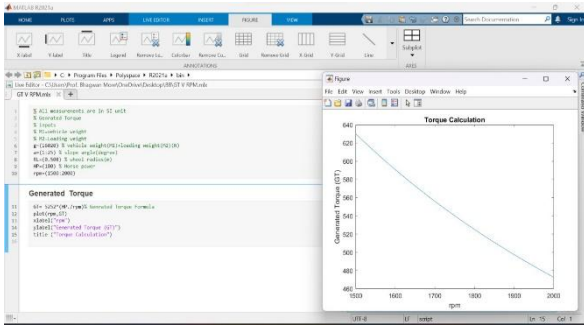
Formula for GT, $GT = 5252 \cdot (\text{HP}/\text{rpm})$

For generated torque, $g = 16020$, $a = 1-25$, $R_L = 0.508\text{m}$, $HP = 180$ rpm range = 1500-250



MATLAB Simulation for Braking Force at 15 Deg

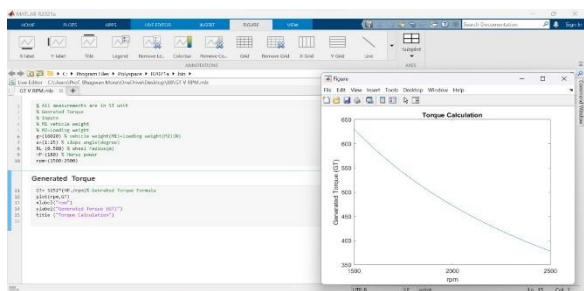




MATLAB Simulation for Generated Torque

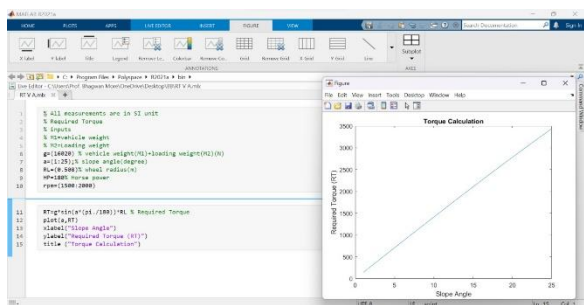
E. MATLAB Simulation for Required Torque

For range of angle 1-25 degree,
wheel Radius (RL) =0.508m, G=16020kg, Rpm range =
1500-2000 Required Torque Formula – $RT = g \cdot \sin(a \cdot (\pi/180)) \cdot RL$

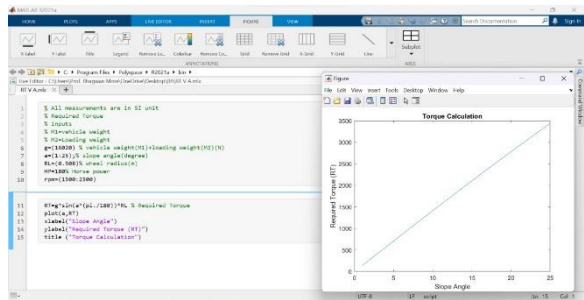


MATLAB Simulation for Generated Torque

For range of angle 1-25 degree,
wheel Radius (RL) =0.508m, G=16020kg, Rpm range =
1500-2500 Required Torque Formula – $RT = g \cdot \sin(a \cdot (\pi/180)) \cdot RL$



MATLAB Simulation for Required Torque (2000)



MATLAB Simulation for Required Torque (2500)

VI. CONCLUSION

This paper has developed a model to predict the brake force at different inclination angles. The focus of the current study

is primarily on manually transverse vehicles equipped with ABS. The vehicle's payload weight, starting speed, stopping distance, and inclination angle were among the input parameters for the mathematical modeling. These could be obtained from the sensors in the vehicle's existing ABS. Previously, assumptions such as Equally Disturbed Mass of Vehicle, 2WD, and Hill Hold Time Interval (3 sec.) were made to make the system more realistic. There was a striking similarity between the computed values and the MATLAB simulation results. Future research will have as its goal evaluating the need for additional sensors, if any, and validating the simulation's findings on a real vehicle.

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been sponsored or funded by any organization or agency. The independence of this research is a crucial factor in affirming its impartiality, as it has been conducted without any external sway.
- **Ethical Approval and Consent to Participate:** The data provided in this article is exempt from the requirement for ethical approval or participant consent.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Authors Contributions:** The authorship of this article is contributed equally to all participating individuals.

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Mathematical Modelling and Analysis to Derive Optimum Brake Pressure for Hill Start Assist System in Commercial Vehicles

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